Radiative Decays $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$ in Effective Field Theory

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Abstract

Heavy hadron chiral perturbation theory (HHχPT) and XEFT are applied to the decays $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$ under the assumption that the $X(3872)$ is a molecular bound state of neutral charm mesons. In these decays the emitted photon energies are 181 MeV and 165 MeV, respectively, so HHχPT can be used to calculate the underlying $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*} \to \psi(2S)\gamma$ or $\psi(4040) \to (D^0\bar{D}^{0*} + \bar{D}^0 D^{0*})\gamma$ transition. These amplitudes are matched onto XEFT to obtain decay rates. The decays receive contributions from both long distance and short distance processes. We study the polarization of the $\psi(2S)$ in the decay $X(3872) \to \psi(2S)\gamma$ and the angular distribution of $X(3872)$ in the decay $\psi(4040) \to X(3872)\gamma$ and find they can be used to differentiate between different decay mechanisms as well as discriminate between 2^{-+} and 1^{++} quantum number assignments of the $X(3872)$.

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The $X(3872)$ [1–3] is the first of many recently discovered hadrons containing hidden charm that do not fit neatly into the traditional model of charmonia as nonrelativistic bound states of $c\bar{c}$. The extreme closeness of the $X(3872)$ to the $D^0\bar{D}^{0*}$ threshold has prompted many authors to suggest that the $X(3872)$ is a molecular bound state of neutral charm mesons, though other possibilities including tetraquark interpretations have also been considered in the literature. For reviews of the recent discoveries in charmonium spectroscopy, see Refs. [4–6].

In this paper, we will work under the assumption that the $X(3872)$ is a shallow S-wave bound state of $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$ and calculate the radiative decays $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$. The interpretation of the $X(3872)$ as a charm meson molecule is motivated by the following considerations: The observed branching ratios [7]

$$\frac{\Gamma[X(3872) \to J/\psi\pi^+\pi^-\pi^0]}{\Gamma[X(3872) \to J/\psi\pi^+\pi^-]} = 1.0 \pm 0.4 \pm 0.3\,, \tag{1}$$

and [8]

$$\frac{\Gamma[X(3872) \to J/\psi\omega]}{\Gamma[X(3872) \to J/\psi\pi^+\pi^-]} = 0.8 \pm 0.3\,, \tag{2}$$

indicate that the $X(3872)$ couples with nearly equal strength to $I = 0$ and $I = 1$ final states. This rules out a conventional charmonium interpretation. The observation of the decay $X(3872) \to J/\psi\gamma$ demands $C = +1$ and the invariant mass distribution in the decay $X(3872) \to J/\psi\pi^+\pi^-$ is consistent with the quantum number assignments $J^{PC} = 1^{++}$ or 2^{-+} only. The decays $X(3872) \to D^0\bar{D}^0\pi^0$ and $X(3872) \to \psi(2S)\gamma$ would suffer an angular-momentum suppression if the $J^{PC} = 2^{-+}$ assignment is correct, leading to a preference for $J^{PC} = 1^{++}$. If the quantum numbers of the $X(3872)$ are 1^{++}, then the $X(3872)$ has an S-wave coupling to the $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$. Finally, since the mass of the $X(3872)$ is 0.42 ± 0.39 MeV below the $D^0\bar{D}^{0*}$ threshold [9], the $X(3872)$ can mix strongly with $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$ and the long range part of the $X(3872)$ wavefunction should be dominated by the $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$ state. Recently, the Babar collaboration studied the three-pion mass distribution in the decay $X(3872) \to J/\psi\pi^+\pi^-\pi^0$ and concluded that the shape prefers the 2^{-+} assignment over 1^{++} [8]. However, the significance of their result is not so great that the 1^{++} assignment can be ruled out. The 2^{-+} assignment is problematic from the point of view of both the conventional quark model as well other interpretations, for discussions see Refs. [10–12]. For the majority of this paper we will assume the 1^{++} assignment for the $X(3872)$ but we will also consider the implications of the 2^{-+} assignment for the radiative decays we will calculate below. An important point of this paper is that these observables may be able to discriminate between the 1^{++} and 2^{-+} quantum number assignments.

If the $X(3872)$ is indeed a shallow bound state of neutral charm mesons, then one can exploit the universal behavior of shallow bound states to compute many $X(3872)$ properties. Universal quantities are those which depend only on the asymptotic form of the bound state wavefunction and known properties of the constituents in the bound state. Examples of this for the $X(3872)$ include the decay rates $\Gamma[X(3872) \to D^0\bar{D}^0\gamma]$ and $\Gamma[X(3872) \to D^0\bar{D}^0\pi^0]$, first calculated by Voloshin in Refs. [13, 14]. In the $X(3872)$, the wavefunction of the $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}$ at a distance much greater than R, where R is the range of the interaction between the charm mesons, takes on the form dictated by quantum mechanics,

$$\psi_{DD^*}(r) \propto \frac{e^{-\gamma r}}{r}\,, \tag{3}$$

where $\gamma = \sqrt{2\mu_{DD^*}B}$, μ_{DD^*} is the reduced mass of the D^0 and $\bar{D}^{0*}$, and B is the binding energy. From the known binding energy, $B = 0.42 \pm 0.39$ MeV, we infer a mean separation $r_X = 4.9^{+13.4}_{-1.4}$ fm, which is incredibly large compared to all known hadrons.

Effective field theory offers a systematic approach to understanding the $X(3872)$ as a molecule. The interactions of the theory are constrained by heavy quark and chiral symmetry via heavy hadron chiral perturbation theory (HHχPT) [16–18]. XEFT [15] is a low energy effective field theory of nonrelativistic D^0, D^{0*}, $\bar{D}^0$, $\bar{D}^{0*}$, and π^0 mesons near the $D^0\bar{D}^{0*}$ threshold that is obtained from HHχPT by integrating out virtual states whose energies are widely separated from the $D^0\bar{D}^{0*}$ threshold. At leading order (LO) XEFT reproduces the universal predictions that follow from the wavefunction in Eq. (3).

In Ref. [19] elastic $D^{(*)}X(3872)$ scattering was calculated using XEFT, and recently Ref. [20] applied XEFT to inelastic $\pi^+X(3872)$ scattering. Both these leading order calculations make predictions which depend only on the binding energy of the $X(3872)$ and known properties of charm mesons with no other undetermined parameters. XEFT can also be used to systematically calculate corrections to universal predictions from effective range corrections, other effects due to higher dimension operators in the XEFT Lagrangian, and corrections from pion loops. In Ref. [15], XEFT was used to calculate corrections to effective range theory predictions for the process $X(3872) \to D^0\bar{D}^0\pi^0$. It was shown that corrections from pion loops were quite small, justifying a perturbative treatment of pions in XEFT.

Finally, XEFT can be used to analyze properties that are not universal but depend on short distance aspects of the $X(3872)$. Here, one seeks factorization theorems for decay rates and cross sections which separate long distance from short distance scales in the $X(3872)$. Factorization theorems for $X(3872)$ production and decay were first obtained in Refs. [21–23]. In XEFT these theorems are obtained by matching HHχPT amplitudes onto XEFT operators, then using these operators to calculate decays and production cross sections in XEFT. An example is the calculation of the hadronic decays $X(3872) \to \chi_J\pi^0$ and $X(3872) \to \chi_J\pi\pi$ [24]. These decays are interesting because the relative rates to final states with different χ_{cJ} can be predicted using heavy quark symmetry [25].

In this paper, we apply XEFT to the radiative decays $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$. The BaBar collaboration quotes the branching fraction [37]:

$$\frac{\Gamma[X(3872) \to \psi(2S)\gamma]}{\Gamma[X(3872) \to J/\psi\gamma]} = 3.4 \pm 1.4\,. \tag{4}$$

Later Belle searched for the decay $X(3872) \to \psi(2S)\gamma$ but did not observe it and obtained an upper bound for the branching ratio in Eq. (4) of 2.1 with a confidence level of 90%. This is consistent with Eq. (4) given the uncertainties, but suggests the true value may be lower than the central value in Eq. (4). Ref. [37] concluded that their measurement disfavored a molecular interpretation of the $X(3872)$, largely because the branching ratio in Eq. (4) was predicted to be 3.7×10^{-3} in the specific molecular model of the $X(3872)$ in Ref. [27]. However, the ratio is sensitive to short-distance components of the $X(3872)$ wavefunction which may not be modelled correctly in the model of Ref. [27]. Ref. [28] describes a model of the $X(3872)$ as a mixed molecule-charmonium state that can account for the branching ratio in Eq. (4).

XEFT alone will not yield a prediction for the branching fraction in Eq. (4) . Since the charm mesons must come to a point to coalesce into a quarkonium, each absolute decay rate in the ratio is sensitive to short distance physics not described by XEFT. Typically one would want to calculate ratios in which this short distance component cancels. But the

$\psi(2S)$ and J/ψ are members of different heavy quark multiplets, with couplings unrelated by symmetry. Finally, the photon energy in the decay $X(3872) \to \psi(2S)\gamma$ is 181 MeV, which is within the range of applicability of HHχPT, while the photon energy in the decay $X(3872) \to J/\psi\gamma$ is 697 MeV, well outside the range of HHχPT. So instead we will analyze what HHχPT and XEFT *can* tell us about $X(3872) \to \psi(2S)\gamma$. We find that there are two distinct mechanisms for the decay $X(3872) \to \psi(2S)\gamma$ and that the polarization of $\psi(2S)$ will shed light on the relative importance of these mechanisms. The polarization is calculated under both the J^{PC} $=1^{++}$ and 2^{-+} assumptions for the quantum numbers of the $X(3872)$ and we discuss how this might be used to distinguish between them.

Another decay that can be analyzed in XEFT is $\psi(4040) \to X(3872)\gamma$, in which the photon energy is 164 MeV. It may be possible to observe this decay at an e^+e^- collider experiment such as BES III if the energy is tuned to the $\psi(4040)$ resonance. The angular distribution (relative to the beam axis) of the $X(3872)$ produced in the process $e^+e^- \to \psi(4040) \to X(3872)\gamma$, yields similar information about the $X(3872)$.

I. $X(3872) \to \psi(2S)\gamma$

The procedure for calculating $X(3872)$ decays to charmonium is described in detail in Ref. [24]. For the decay $X(3872) \to \psi(2S)\gamma$, one first calculates the transition amplitude for $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*} \to \psi(2S)\gamma$ using HHχPT, extended to include charmonium states as explicit degrees of freedom. HHχPT Lagrangians with quarkonia were first developed in Refs. [30, 31]. For a recent application to radiative decays of quarkonia, see Ref. [32]. These papers used a covariant formulation in which the heavy mesons in the initial and final states can have distinct four-velocities. We will use the two-component version of HHχPT introduced in Ref. [34]. This formalism uses two-component spinors with the four velocity for both the initial and final heavy mesons fixed to be $v^\mu = (1, \vec{0})$. This formalism is suitable for processes in which the recoil of the heavy particle in the final state can be neglected, which is the case for this decay since $v_i \cdot v_f = (m_X^2 + m_\psi^2)/(2m_X m_\psi) = 1.001$, where $v_i(v_f)$ denotes the four-velocity of the initial (final) quarkonium.

The interaction Lagrangian for $X(3872) \to \psi(2S)\gamma$ is given by

$$\begin{aligned}\mathcal{L} \;=\; & \frac{e\beta}{2}\mathrm{Tr}[H_1^\dagger H_1\, \vec{\sigma}\cdot\vec{B}\, Q_{11}] + \frac{eQ'}{2m_c}\mathrm{Tr}[H_1^\dagger\, \vec{\sigma}\cdot\vec{B}\, H_1] + h.c. \\ & + i\frac{g_2}{2}\mathrm{Tr}[J^\dagger H_1 \vec{\sigma}\cdot \overleftrightarrow{\partial}\, \bar{H}_1] + i\frac{ec_1}{2}\mathrm{Tr}[J^\dagger H_1 \vec{\sigma}\cdot\vec{E}\bar{H}_1] + h.c.\,. \end{aligned} \tag{5}$$

Here $Q_{11} = \frac{2}{3}$, $Q' = \frac{2}{3}$, h.c. means hermitian conjugate, and J is the superfield containing the $\psi(2S)$ and $\eta_c(2S)$. The first two terms contain the couplings of the charm mesons to photons, the third term contains the coupling of the charm mesons to charmonia, and the final (contact) term couples the charm mesons, the charmonia, and the electric field. While g_2 and c_1 are unknown parameters at present, β occurs in HHχPT predictions involving measured quantities: Ref. [33] obtains $\beta^{-1} \sim 1200$ MeV from radiative decays within the lowest charm meson multiplet, Ref. [34] found $\beta^{-1} = 275 - 375$ MeV, and Ref. [35] included the effects of the excited charm meson multiplet to find $\beta^{-1} = 670$ MeV. Since we have integrated the excited charm mesons out and neglected loop corrections in the HHχPT calculations in this paper, we will use the value of $\beta^{-1} = 275 - 375$ MeV extracted in Ref. [34], which makes the same approximations. From these interactions we find four tree-level diagrams contributing to $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*} \to \psi(2S)\gamma$, which are shown in Figs. 1a)-d).

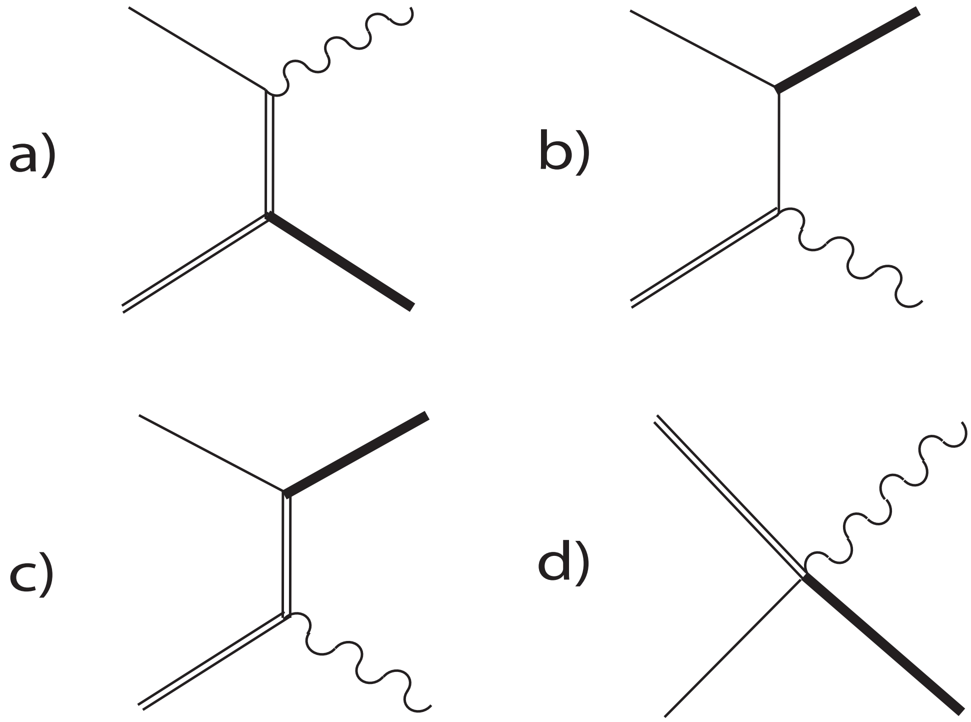

FIG. 1: Feynman diagrams contributing to the $D^0\bar{D}^{0*} \to \psi(2S)\,\gamma$ amplitude. The thin solid line is a D^0 meson, the double line is a $\bar{D}^{0*}$ meson, the wavy line is a photon, and the thick solid line is the $\psi(2S)$.

The amplitudes corresponding to each of these diagrams are

$$a) \;=\; -\frac{g_2\, e\, \beta_+}{3}\frac{1}{E_\gamma + \Delta}(\vec{k}\cdot\vec{\epsilon}_\psi^{\,*}\,\vec{\epsilon}_{D^*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*} - \vec{k}\cdot\vec{\epsilon}_{D^*}\,\vec{\epsilon}_\psi^{\,*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*}) \tag{6}$$

$$b) \;=\; \frac{g_2\, e\, \beta_+}{3}\frac{1}{\Delta - E_\gamma}\,\vec{k}\cdot\vec{\epsilon}_\psi^{\,*}\,\vec{\epsilon}_{D^*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*} \tag{7}$$

$$c) \;=\; \frac{g_2\, e\, \beta_-}{3}\frac{1}{E_\gamma}\,\vec{k}\cdot\vec{\epsilon}_{D^*}\,\vec{\epsilon}_\psi^{\,*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*} \tag{8}$$

$$d) \;=\; -e\, c_1\, E_\gamma\,\vec{\epsilon}_{D^*}\cdot\vec{\epsilon}_\psi^{\,*}\times\vec{\epsilon}_\gamma^{\,*}\,, \tag{9}$$

where $\beta_\pm = \beta \pm 1/m_c$, the polarization vectors of the photon, D^{0*}, and $\psi(2S)$ are $\vec{\epsilon}_\gamma^{\,*}, \vec{\epsilon}_{D^*}$, and $\vec{\epsilon}_\psi^{\,*}$, respectively, and $\vec{k}$ is the outgoing photon momentum..

An additional potential contribution to $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*} \to \psi(2S)\gamma$ is $D^0\bar{D}^{0*} + \bar{D}^0 D^{0*} \to \chi_{c1}(2P) \to \psi(2S)\gamma$. It is quite likely that the masses of the $\chi_{cJ}(2P)$ states are close to the $X(3872)$ mass. For example, Ref. [36] quotes quark model predictions for the $\chi_{c1}(2P)$ mass of 3925 MeV (in a nonrelativistic potential model) and 3953 MeV (in the Godfrey-Isgur relativistic quark model). Alternatively, if the $Z(3930)$ is the $\chi_{c2}(2P)$ state one expects the $\chi_{c1}(2P)$ to be about 3885 MeV, assuming that the spin-orbit splitting for $\chi_{cJ}(2P)$ states is equal to the observed spin-orbit splitting for $\chi_{cJ}(1P)$ states. (The nonrelativistic potential model predicts this splitting to be approximately the same, while the Godfrey-Isgur model predicts it to be slightly smaller.) In this scenario, the $\chi_{c1}(2P)$ is within 14 MeV of the $X(3872)$ and the process $D^0\bar{D}^{0*} \to \chi_{c1}(2P) \to \psi(2S)\gamma$ could be important for the radiative decay of the $X(3872)$.

The decay $\chi_{cJ} \to \psi(2S)\gamma$ is an electric dipole transition mediated by the operator

$$\mathcal{L} = \delta^{2P2S} \mathrm{Tr}[J^\dagger \chi_c^i] E^i + h.c. , \tag{10}$$

where E^i is the electric field, χ_c is the super field containing the $\chi_{cJ}(2P)$ states, and the coupling constant δ^{2P2S} is the same as the one defined in Ref. [32], which calculated the decay rate

$$\Gamma[\chi_{c1}(2P) \to \psi(2S)\gamma] = \frac{(\delta^{2P2S})^2}{3\pi} \frac{m_{\psi(2S)}}{m_{\chi_{c1}(2P)}} k_\gamma^3 . \tag{11}$$

The charm mesons couple to the $\chi_{cJ}(2P)$ through a coupling

$$\mathcal{L} = \frac{i}{2} g_1' \, \mathrm{Tr}[\chi_c^{\dagger i} \bar{H} \sigma^i H] + h.c. , \tag{12}$$

This coupling is exactly the same as the coupling of heavy mesons to $\chi_{cJ}(1P)$ states introduced in Ref. [24], except now the χ_c^i superfield contains the $\chi_{cJ}(2P)$ states and the coupling is g_1' instead of g_1. The effect of including a tree-level diagram for $D^0 \bar{D}^{0*} + \bar{D}^0 D^{0*} \to \psi(2S)\gamma$ using the vertices in Eqs. (10) and (12) is to modify amplitude $d)$ in Eq. (6) by the substitution

$$e c_1 \to e c_1 + \frac{g_1' \delta^{2P2S}}{m_X - m_{\chi_{c1}(2P)}} . \tag{13}$$

At present, δ^{2P2S}, g_1', and $m_{\chi_{c1}(2P)}$ are unknown, so in what follows we will simply absorb this contribution into the definition of the coupling c_1.

An illuminating observable is the decay rate for $X(3872) \to \psi(2S)(\vec{\epsilon}_\psi)\gamma$, where the polarization vector $\vec{\epsilon}_\psi$ of the produced $\psi(2S)$ can in principle be determined from the angular distribution of the leptons into which it decays: $\psi(2S) \to \ell^+\ell^-$. Averaging over the initial $X(3872)$ and final photon polarizations we find

$$\begin{aligned} \Gamma[X(3872) \to \psi(2S)(\vec{\epsilon}_\psi)\gamma] \; &= \; \sum_\lambda |\langle 0| \frac{1}{\sqrt{2}} \epsilon^i(\lambda) \, (V^i \, \bar{P} + \bar{V}^i \, P) |X(3872, \lambda)\rangle|^2 \\ &\times \frac{m_\psi}{m_X} \frac{E_\gamma}{24\pi} \left(\frac{2}{3} \, (A + C)^2 \; |\hat{k} \cdot \vec{\epsilon}_\psi|^2 + \frac{1}{3} \, (B - C)^2 \, |\hat{k} \times \vec{\epsilon}_\psi|^2 \right) , \end{aligned} \tag{14}$$

where V^i and P are the vector and scalar components of the $D^{(*)}$ superfield, and $\epsilon^i(\lambda)$ are a basis of polarization vectors for the $X(3872)$. In Eq. (14), $\hat{k}$ is a unit vector in the direction of the photon's three-momentum, and

$$A = \frac{g_2 e \beta_+}{3} \frac{2 E_\gamma^3}{\Delta^2 - E_\gamma^2} \quad B = \frac{g_2 e}{3} \frac{\beta_+ E_\gamma^2 + \beta_- E_\gamma (E_\gamma + \Delta)}{E_\gamma + \Delta} \quad C = - e c_1 E_\gamma . \tag{15}$$

We have used $\vec{\epsilon}_\psi^{\,*} \cdot \vec{\epsilon}_\psi = |\hat{k} \cdot \vec{\epsilon}_\psi|^2 + |\hat{k} \times \vec{\epsilon}_\psi|^2$. The total decay rate is given by

$$\begin{aligned} \Gamma[X(3872) \to \psi(2S)\gamma] \; &= \; \sum_\lambda |\langle 0| \frac{1}{\sqrt{2}} \epsilon_i(\lambda) \, (V^i \, \bar{P} + \bar{V}^i \, P) |X(3872, \lambda)\rangle|^2 \\ &\times \frac{E_\gamma}{36\pi} \frac{m_\psi}{m_X} \left[(A + C)^2 + (B - C)^2 \right] . \end{aligned} \tag{16}$$

In addition to not having an experimental determination of the parameters g_2 and c_1 contained in A, B, and C, the matrix element in Eq. (16) is unknown; additional measurements will be necessary to make a prediction for the total rate. However, the matrix element between $X(3872)$ and its constituents appears in any process involving the $X(3872)$, so a measurement from a different production or decay chain can be used in this calculation. Combining the lower bound $\Gamma[X(3872) \to \psi(2S)\gamma]/\Gamma[X(3872)] > 3.0 \times 10^{-2}$ from Refs. [37, 38] with the upper bound on the total width $\Gamma[X(3872)] < 2.3$ MeV [1] yields the lower bound on the partial width $\Gamma[X(3872) \to \psi(2S)\gamma] > 7 \times 10^{-2}$ MeV.

If we define $|\mathcal{M}_\parallel|^2$ ($|\mathcal{M}_\perp|^2$) to be the matrix element squared for decay into $\psi(2S)$ polarized parallel (perpendicular) to the axis defined by the photon momentum, then

$$\begin{aligned} |\mathcal{M}_\parallel|^2 &= \frac{2}{3}(A+C)^2 \\ |\mathcal{M}_\perp|^2 &= \frac{2}{3}(B-C)^2 \,. \end{aligned} \tag{17}$$

It is interesting to consider the limits i) $|g_2\beta_\pm| \ll |c_1|$ and ii) $|g_2\beta_\pm| \gg |c_1|$. When $|g_2\beta_\pm| \ll |c_1|$ the short distance contribution dominates, $|C| \gg |A|, |B|$, and

$$i) \qquad \frac{|\mathcal{M}_\parallel|^2}{|\mathcal{M}|^2} = \frac{|\mathcal{M}_\parallel|^2}{|\mathcal{M}_\parallel|^2 + |\mathcal{M}_\perp|^2} = \frac{1}{2}\,. \tag{18}$$

That is, diagram d) yields $|\mathcal{M}_\parallel|^2 = |\mathcal{M}_\perp|^2$. In case ii), diagrams a) -c) dominate and we find

$$ii) \qquad \frac{|\mathcal{M}_\parallel|^2}{|\mathcal{M}|^2} = \frac{4E_\gamma^4}{4E_\gamma^4 + (E_\gamma + r_\beta(E_\gamma+\Delta))^2(E_\gamma-\Delta)^2} = 0.95\,(0.92)\,. \tag{19}$$

where $r_\beta \equiv \beta_-/\beta_+$. The first number on the r.h.s. of Eq. (19) corresponds to r_β in the range 0.62-0.69, taken from fits in Ref. [34], while the number in parentheses corresponds to $r_\beta = 1$. In case ii) diagrams a)-c) dominate over diagram d), and diagram b) dominates diagrams a)-c) because $E_\gamma - \Delta \sim 39$ MeV is small. The result is that the polarization of the produced $\psi(2S)$ is dictated by diagram b), which peaks for longitudinally polarized $\psi(2S)$. The angular distribution of the final state lepton pair in the decay $\psi(2S) \to \ell^+\ell^-$ is

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha\cos^2\theta \qquad \alpha = \frac{1-3f_L}{1+f_L}\,, \tag{20}$$

where $f_L = |\mathcal{M}_\parallel|^2/|\mathcal{M}|^2$ and $\cos\theta$ is the angle between the lepton's and the photon's momentum. For case i) $\alpha = -1/3$ and for case ii) $\alpha = -0.91(-0.95)$, so the angular distribution of the leptons is sensitive to the production mechanism and can be used to distinguish among them.

Defining $\lambda = 3c_1/(g_2\beta_+)$, $\lambda \to 0$ corresponds to diagrams a)-c) dominating, while $|\lambda| \to \infty$ corresponds to the contact interaction dominating. In terms of λ,

$$f_L = \frac{N}{D}\,,$$

where

$$\begin{aligned} N &= \left(\frac{2E_\gamma^2}{\Delta^2 - E_\gamma^2}\right)^2 - \lambda\,\frac{4E_\gamma^2}{\Delta^2 - E_\gamma^2} + \lambda^2 \\ D &= \left(\frac{2E_\gamma^2}{\Delta^2 - E_\gamma^2}\right)^2 + \left(\frac{E_\gamma + r_\beta(E_\gamma+\Delta)}{E_\gamma+\Delta}\right)^2 - 2\lambda\left(\frac{2E_\gamma^2}{\Delta^2 - E_\gamma^2} - \frac{E_\gamma + r_\beta(E_\gamma+\Delta)}{E_\gamma+\Delta}\right) + 2\lambda^2\,. \end{aligned} \tag{21}$$

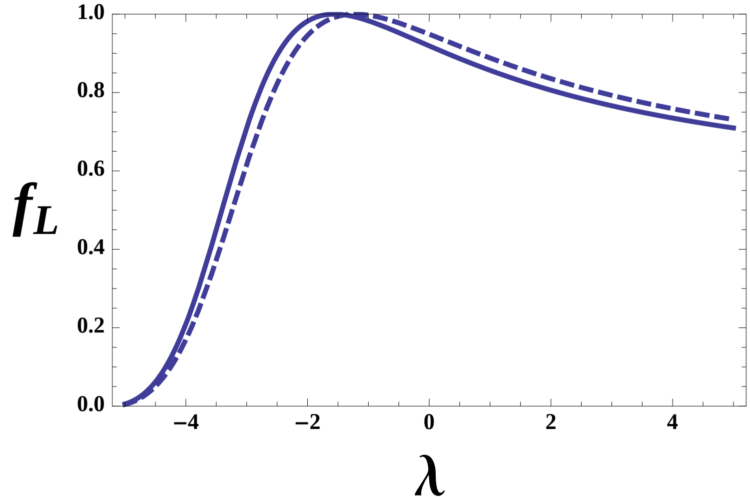

FIG. 2: f_L as a function of the parameter λ (defined in text). Solid line corresponds to $r_\beta = 1.0$, dashed line to $r_\beta = 0.66$.

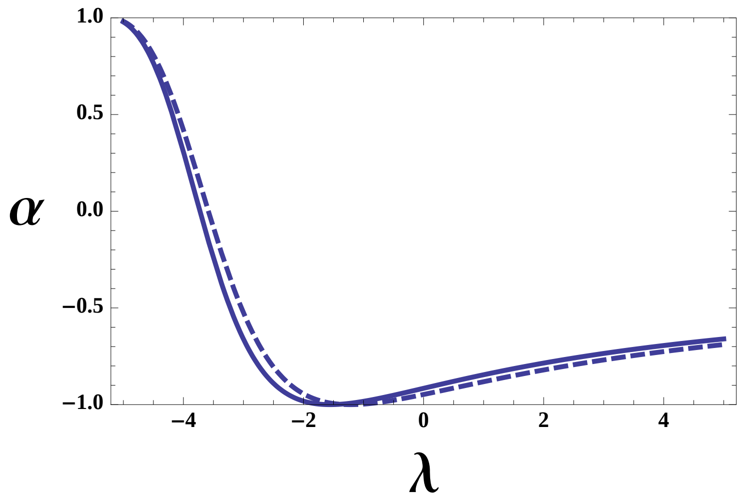

FIG. 3: α as a function of the parameter λ (defined in text). Solid line corresponds to $r_\beta = 1.0$, dashed line to $r_\beta = 0.66$.

Fig. 2 is a plot of f_L as a function of λ and Fig. 3 is a plot of α in terms of the parameter λ. Naive dimensional analysis suggests $\lambda \sim O(1)$, so the plots range over $-5 < \lambda < 5$. The plots show the results for $r_\beta = 1$ (solid) and $r_\beta = 0.66$ (dotted). The behavior shown in the plots remains the same when r_β is varied between $-1 < r_\beta < 1$, where the lower limit corresponds to the situation where the $1/m_c$ term (cf. Eq. (5)) dominates while the upper limit is the heavy quark limit. The curves just continue to move to the right for smaller values of r_β. For the most likely values of r_β, longitudinal polarization ($f_L \geq 1/2$ and $\alpha \leq -1/3$) is found for λ in the range $-3 < \lambda < 5$.

This analysis potentially yields a method for determining the amount of a molecular versus nonmolecular description consistent with a 1^{++} assignment for the $X(3872)$. If the multipole expansion is legitimate, the leading order description of a nonmolecular 1^{++} is a

P-wave contact term equivalent to c_1. So to the extent that the $\psi(2S)$ polarization in the $X(3872) \to \psi(2S)\gamma$ decay is found to be longitudinally polarized, the molecular description dominates its character.

It is also interesting to consider what the $J^{PC} = 2^{-+}$ assignment for the $X(3872)$ would imply for the $\psi(2S)$ polarization. Denote the spin-2 field in HHχPT by X^{ij}, where X^{ij} is symmetric and traceless in its indices. The simplest coupling mediating $X(3872) \to \psi(2S)\gamma$ is

$$\mathcal{L} = g' \, \mathrm{Tr}[X^{ij} J^\dagger \sigma^i] B^j \,, \tag{22}$$

which yields an amplitude proportional to

$$\mathcal{M}[X(3872) \to \psi(2S)(\vec{\epsilon}_\psi)\gamma] \propto \vec{\epsilon}_\psi^{\,*i} (\vec{k} \times \vec{\epsilon}_\gamma^{\,*})^j h^{ij} \,, \tag{23}$$

where $\vec{k}$ is the photon three-momentum, and $\vec{\epsilon}_\psi^{\,*}$, $\vec{\epsilon}_\gamma^{\,*}$, and h^{ij} are the polarization tensors for the $\psi(2S)$, photon, and $X(3872)$, respectively. Summing over the polarizations of the $X(3872)$ and the photon, the cross section's dependence on the polarization of the $\psi(2S)$ becomes

$$\sum |\mathcal{M}[X(3872) \to \psi(\vec{\epsilon}_\psi)\gamma]|^2 \propto |\vec{k} \cdot \vec{\epsilon}_\psi|^2 + \frac{7}{6} |\vec{k} \times \vec{\epsilon}_\psi|^2 \,. \tag{24}$$

The fraction of longitudinally polarized $\psi(2S)$ is $f_L = 0.3$, corresponding to $\alpha = 0.08$. This leading order description of a $J^{PC} = 2^{-+}$ $X(3872)$ yields a very slight transverse polarization of the $\psi(2S)$.

Ref. [10] assumes that the $X(3872)$ is the $\eta_c(^1D_2)$. In the models considered in that paper, the leading contribution to the decay is an M1 amplitude identical in form to that given by Eq. (22). In addition, the models include electric quadrupole and magnetic octopole transitions (which correspond to higher dimension operators in HHχPT). From the helicity amplitudes calculated in the five potential models of Ref. [10], we obtain $f_L = 0.11 - 0.28$ ($\alpha = 0.13 - 0.6$). This suggests the $J^{PC} = 2^{-+}$ quantum number assignment prefers slightly transverse polarization for the $\psi(2S)$ in the $X(3872) \to \psi(2S)\gamma$ decay. In contrast, the molecular (1^{++}) hypothesis predicts longitudinal polarization in much (but not all) of parameter space.

II. $\psi(4040) \to X(3872)\gamma$

Assuming that the $\psi(4040)$ is the 3^3S_1 charmonium, the interaction Lagrangian for $\psi(4040) \to X(3872)\gamma$ is essentially the same as in Eq. (5). The superfield J should be replaced by the superfield containing the $\psi(4040)$ while the couplings g_2 and c_1 are replaced by analogous couplings $\tilde{g}_2$ and $\tilde{c}_1$. The diagrams for $\psi(4040) \to (D^0\bar{D}^{0*} + \bar{D}^0 D^{0*})\gamma$ are related to those in Fig. 1 by crossing symmetry. The corresponding amplitudes are

$$\begin{aligned}
a) &= -\frac{\tilde{g}_2\, e\, \beta_+}{3} \frac{1}{E_\gamma - \Delta} (\vec{k} \cdot \vec{\epsilon}_\psi \, \vec{\epsilon}_{D^*} \cdot \vec{k} \times \vec{\epsilon}_\gamma^{\,*} - \vec{k} \cdot \vec{\epsilon}_{D^*} \, \vec{\epsilon}_\psi \cdot \vec{k} \times \vec{\epsilon}_\gamma^{\,*}) && (25)\\
b) &= -\frac{g_2\, e\, \beta_+}{3} \frac{1}{E_\gamma + \Delta} \vec{k} \cdot \vec{\epsilon}_\psi \, \vec{\epsilon}_{D^*} \cdot \vec{k} \times \vec{\epsilon}_\gamma^{\,*} && (26)\\
c) &= \frac{\tilde{g}_2\, e\, \beta_-}{3} \frac{1}{E_\gamma} \vec{k} \cdot \vec{\epsilon}_{D^*} \, \vec{\epsilon}_\psi \cdot \vec{k} \times \vec{\epsilon}_\gamma^{\,*} && (27)\\
d) &= -e\, \tilde{c}_1\, E_\gamma \, \vec{\epsilon}_{D^*} \cdot \vec{\epsilon}_\psi \times \vec{\epsilon}_\gamma^{\,*} \,. && (28)
\end{aligned}$$

We are interested in the angular distribution of the $X(3872)$ produced in the process $e^+e^- \to \psi(4040) \to X(3872)\gamma$. The mass of the electrons is negligible compared to the $\psi(4040)$; the electrons are treated as helicity eigenstates whose spin angular momentum is projected along the beam axis. Thus the $\psi(4040)$ has $L_z = \pm 1$, where the beam axis defines the z-direction. Therefore the $\psi(4040)$ is produced with polarization normal to the beam axis. This then dictates the angular distribution of the $X(3872)$ produced in the decay. If we square the amplitudes, and average over the $X(3872)$ and γ polarizations, we find the matrix element squared is

$$\sum |\mathcal{M}(\vec{\epsilon}_\psi)|^2 \propto \frac{2}{3} P\,|\hat{k}\cdot\vec{\epsilon}_\psi|^2 + \frac{1}{3} T\,|\hat{k}\times\vec{\epsilon}_\psi|^2 \,, \tag{29}$$

where $\vec{\epsilon}_\psi$ is the $\psi(4040)$ polarization vector, $\hat{k}$ is unit-vector along the three-momentum of the photon in the $\psi(4040)$ rest frame, and P and T are given by:

$$\begin{aligned} P &= \left(\frac{\tilde{g}_2 e\beta_+}{3}\frac{2E_\gamma^3}{\Delta^2 - E_\gamma^2} - e\tilde{c}_1 E_\gamma\right)^2 \\ T &= \left(\frac{\tilde{g}_2 e\beta_+}{3}\frac{E_\gamma^2 + r_\beta E_\gamma(E_\gamma - \Delta)}{E_\gamma - \Delta} + e\tilde{c}_1 E_\gamma\right)^2 \,. \end{aligned} \tag{30}$$

The angular distribution can be obtained by replacing $\epsilon_\psi^i \epsilon_\psi^{*j} = \delta^{ij} - \hat{z}^i\hat{z}^j$ in Eq. (29). Defining θ to be the angle that the $X(3872)$ (or the photon) makes with the beam axis, we find

$$\frac{d\sigma}{d\,\cos\theta} \propto 1 + \rho\cos^2\theta \,, \tag{31}$$

where ρ is given by

$$\rho = \frac{T - 2P}{T + 2P} \,. \tag{32}$$

The value of ρ in Eq. (31) depends on the following combination of HHχPT coupling constants:

$$\Lambda \equiv \frac{3\tilde{c}_1}{\tilde{g}_2\beta_+} \,. \tag{33}$$

Fig. 4 is a plot of ρ as a function of the dimensionless parameter Λ, for $-10 \leq \Lambda \leq 10$. Λ is expected to be $O(1)$. In the region where $\tilde{c}_1$ dominates, $|\Lambda| \to \infty$, and ρ asymptotes to $-1/3$. As r_β decreases, ρ reaches the asymptote at larger values of Λ. Near $\Lambda \sim -8$, ρ is very sensitive to Λ and can take on any value between -1 and $+1$. For comparison, if the $X(3872)$ has quantum numbers $J^{PC} = 2^{-+}$ and couples to the $\psi(4040)$ and the photon by the leading order operator analogous to Eq. (22), $\rho = 1/13 = 0.08$. So the angular distribution of $X(3872)$ produced in the process $e^+e^- \to \psi(4040) \to X(3872)\gamma$ can also be used to discriminate between quantum number assignments of the $X(3872)$.

III. SUMMARY

In this paper we have calculated the radiative decays $X(3872) \to \psi(2S)\gamma$ and $\psi(4040) \to X(3872)\gamma$ using XEFT. Each receives contributions from a "long-distance" portion involving

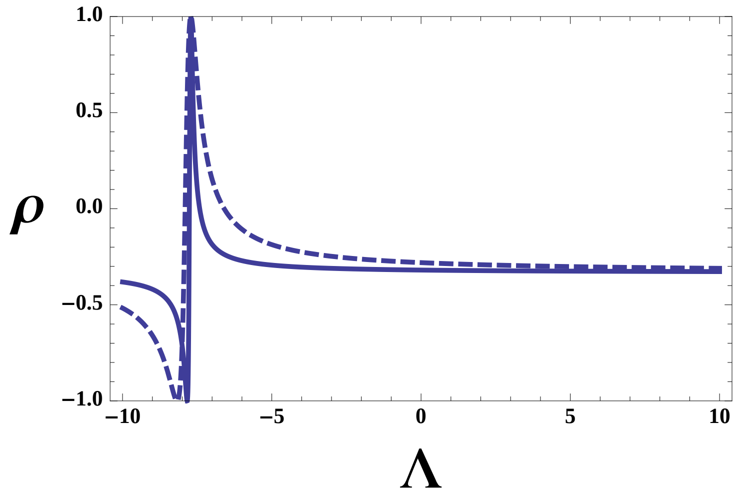

FIG. 4: The parameter ρ from the angular distribution of $X(3872)$ in the decay $\psi(4040) \to X(3872)\gamma$ as a function of $\Lambda \equiv 3\tilde{c}_1/(\tilde{g}_2\beta_+)$. The solid line has $r_\beta = 1.0$ and the dashed line has $r_\beta = 1.0$.

the propagation of a heavy charm meson (diagrams a)-c) in Fig. (1)), and a short-distance contact operator (diagram d) in Fig. (1)). The relative importance of these two types of diagrams depends on the ratio of two undetermined parameters in the HHχPT Lagrangian; λ for the $X(3872)$ decay mechanism above and Λ for the $X(3872)$ production mechanism. A primary result of this paper is that the angular distributions of decay products can be used to distinguish between the 1^{++} and 2^{-+} assignments of the $X(3872)$ as well as the relative importance of the two types of diagrams involved. The polarization of the $\psi(2S)$ produced in the decay $X(3872)_{1^{++}} \to \psi(2S)\gamma$ is sensitive to λ. In much of the parameter space the $\psi(2S)$ is longitudinally polarized. In contrast, for $X(3872)_{2^{-+}} \to \psi(2S)\gamma$, $\psi(2S)$ is produced with a slight transverse polarization. A similar set of diagrams to those in Fig. (1)) (with different coupling constants) contributes to the decay $\psi(4040) \to X(3872)\gamma$. In the process $e^+e^- \to \psi(4040) \to X(3872)\gamma$, the angular distribution of the $X(3872)$ (or γ) relative to the e^+e^- beam axis can discriminate between the 1^{++} and 2^{-+} assignments of $X(3872)$. In most of parameter space, the parameter ρ in Eq. (31) is near $-1/3$ for $X(3872)_{1^{++}}$, while $X(3872)_{2^{-+}}$ produces $\rho \approx 0.08$.

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